One-shot lotteries in the park

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Abstract

How do people manipulate their environment when balancing trade-offs between probability of success and payoff? Individuals in a city park played a simple lottery using a small set of marbles placed in an urn. Participants had the ability to actively improve their chances of winning but only by reducing the amount of money that they could possibly win. Hence, participants controlled the lottery's intuitive trade-off between probability of success and potential payout. Across four different lottery structures, participants, on average, behaved systematically *safer* than the optimal strategy that maximizes expected gain. We explore two different accounts of this suboptimal choice behavior: probability distortion, and intrinsic utility of winning.

Keywords: decision making; one-shot lottery; probability distortion; intrinsic utility of winning.

Introduction

Typical decision making studies offer participants choices between two fixed alternatives. Sometimes the prospects are explicitly described: "Would you prefer to draw from Deck-1 which awards $2 with .8 probability or from Deck-2 which awards $6 with .3 probability?" Other times the prospects must be learned from experience: "After sampling repeatedly from both decks and observing the outcomes, you will choose which deck to draw from for the trial that counts." Dissimilar results from these two kinds of experimental paradigms have led to an explosion of research concerning the description-experience gap in decision-making under risk (for a brief review, see Hertwig & Erev, 2009).

While this dichotomy has gathered considerable focus, some decisions are not clearly descriptive or clearly experience-based. We think of these situations as intuitive "everyday" decisions that implicitly select one out of many choices available. A key feature of such decisions is that people need not consider explicitly every possible alternative to make a decision.

In the present study, participants manipulated their environment to choose between a large number of different prospects that weren't explicitly described and weren't directly experienced. The experimental task was a one-shot lottery whose parameters (probability of winning and magnitude of monetary prize) were partially under participants' control. Critically, the lottery was carefully designed so that increasing the probability of winning automatically decreased the potential monetary prize, and increasing the potential monetary prize automatically decreased the probability of winning (an inverse relationship typical of many real-world lotteries).

Our goal in this study was three fold. First, we were interested in how people approach situations where they have control over a potentially rewarding stochastic environment (see also Juni, Gureckis, & Maloney, 2011). In particular, do people manipulate their environment to maximize their expected gain? Second, we were interested in intuitive "everyday" decision-making where a large number of prospects must be discerned. Finally, we were interested in taking some of our recent decision making research out of the laboratory to consider a more diverse population of decision makers.

Taking decision research out of the lab

The majority of psychological research in cognitive science on decision making is conducted using laboratory studies with college undergraduates. However, a number of recent arguments have been presented for why such populations may not be representative of the general human population (Henrich, Heine, & Norenzayan, 2010). In addition, a large percentage of decision making research is conducted in the laboratory on computers for either real or hypothetical amounts of money. One concern about computer-based studies is that participants may suspect that the odds or payouts are being manipulated as part of the study design.

To address these concerns, we ventured outside of the laboratory and into the streets of a large US city to elicit choice behavior from randomly chosen pedestrians who were posed with a single, non-hypothetical problem that was played out for real, in person. Those who agreed to participate were informed that the lottery would be played only once and that they would receive real money if they won. The lottery was performed using a physical urn and marbles that the participant could see and touch. This guaranteed that participants could be certain that there was no manipulation of the odds (e.g., by a computer program).

The "marbles" game

The one-shot lottery we implemented is very intuitive. The urn initially contains several black marbles and no white marbles. After agreeing to participate, the subject is handed several white marbles. To win the lottery, the participant must, without looking, pull out a white marble from the urn. Thus, the participants must put at least some of the white marbles into the urn so that they have a chance of winning the lottery.

Of course, one strategy might be to place all the white marbles in the urn (to maximally increase the odds of successfully drawing a white marble). However, the monetary prize for pulling out a white marble from the urn is determined by the number of white marbles that the participant chooses to *not* put into the urn but rather set aside as potential prize money.

Each white marble that was not placed into the urn represents $1 in prize money, which is only awarded if the marble that the participant pulls from the urn turns out to be white.

Four different conditions were tested which varied the number of black marbles that were first placed in the urn: 1, 2, 8, or 25. In all conditions the participant was handed 10 white marbles. If the participant were to put zero white marbles into the urn, they have no chance of winning. On the other hand, if they were to put all 10 white marbles into the urn, they win nothing even if they draw a white marble. Thus, the experimental design restricted participants to put anywhere between 1 and 9 white marbles into the urn. This number becomes our primary dependent measure (i.e., "how many white marbles do participants put into the urn as a function of the number of black marbles in the urn?").

Figure 1 shows how the different probability structures of the four conditions affects the respective expected gain functions. The figure also shows the respective number of white marbles that should be put into the urn to maximize expected gain in each condition. As is visible, the lottery was deliberately designed so that the normative ideal rule regarding how many white marbles should be put into the urn is different for each condition. When there is one black marble in the urn, the participant can maximize expected gain by putting two white marbles into the urn. When there are two black marbles, expected gain is maximized by putting three whites marbles into the urn. When there are eight black marbles, expected gain is maximized by putting four white marbles into the urn. And, finally, when there are 25 black marbles in the urn[1], expected gain is maximized by putting five white marbles into the urn.

The basic question asked in our study is if people combine information about probability and reward to maximize their expected gain in this intuitive one-shot lottery.

Methods

Subjects Data was collected from people walking through Washington Square Park in New York City. 120 people (65 males and 55 females) participated in the experiment (30 per condition). Ages ranged from 18 to 75 years, with an average age of 29.77 years (SD=14.65) and a median age of 24 years. Participants were not compensated for their time, but they did receive the prize money (anywhere between $1 and $9) if they won their lottery.

Materials To conduct the lottery we used transparent cups, an opaque urn, 10 white marbles, and a varying number of black marbles (1, 2, 8, or 25). The black and white marbles were identical except for color and could not be identified through touch.

[1] In the limit, as the number of black marbles grows very large, the rule that maximizes expected gain is to put half of the white marbles into the urn and set the other half aside as prize money. Given that participants were given 10 white marbles to work with, it is impossible for the optimal rule to dictate placing more than five of them into the urn no matter how many black marbles there are in the urn. We thank Hang Zhang for pointing this out to us.

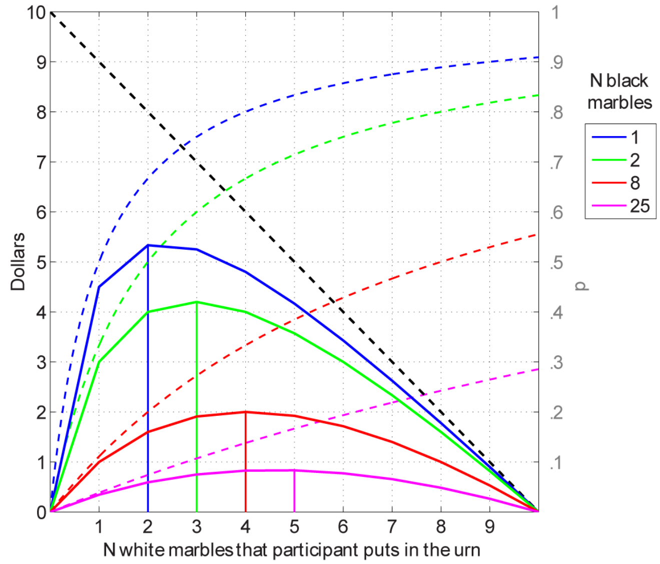

Figure 1: Experimental design. The participant decides how many white marbles to put into the urn and how many to set aside as the potential prize for winning the lottery. The black dashed diagonal shows the prize money that the participant receives if the marble that she pulls out from the urn turns out to be white. It starts at $9 when only one white marble is put in the urn and declines to $1 when nine white marbles are put into the urn. The colored dashed curves show the probability (p) of winning the lottery as a function of the number of black marbles in the urn and the number of white marbles that the participant chooses to put into the urn. Each probability function is multiplied by the gain function (i.e., the black dashed diagonal) to generate an expected gain function. The colored solid lines show the discrete expected gains of the lottery as a function of the number of black marbles in the urn and the number of white marbles that the participant chooses to put into the urn. The vertical colored lines show the maximum expected gain for each of the four experimental conditions.

Procedure The lottery was conducted next to a bench in Washington Square Park. On the bench there was an empty urn, a cup with white marbles, and a cup with black marbles. The experimenter handed the participant the cup with the white marbles and said the following:

"To conduct the lottery we will be using marbles. You are in control of the white marbles. There are 10 white marbles and each represents $1. I am in control of the black marbles. There are (1, 2, 8, 25) of them."

The experimenter then showed the participant an envelope with a large number $1 bills and emphasized that if the participant won the lottery they would receive real money. Next, the experimenter poured the black marbles into the urn and said the following:

"I have poured the black marbles into the urn. To perform the lottery, you will be placing your hand into the urn and pulling out a single marble without looking. If the marble that comes out is black you win nothing. If the marble that comes out is white you will win the number of white marbles that you chose not to put into the urn but rather set aside as your potential prize money. This lottery will be performed only once."

Next the experimenter held the urn in one hand and an empty cup in the other hand and said the following:

"Take your white marbles and decide how many you want to put into the urn for the lottery and how many you want to set aside in this cup as your potential prize money if you win the lottery. Remember, each white marble is worth $1."

Once the participant divided up the white marbles between the urn and the prize cup, the experimenter asked the participant to confirm verbally what the monetary prize would be if they pulled out a white marble from the urn to ensure that there weren't any misunderstandings.

Next the experimenter held up the urn above the participant's eyes and asked the participant to pull out a single marble from the urn. If the participant pulled out a black marble they received no money; if they pulled out a white marble they were paid $1 for each white marble that they set aside in the prize cup.

Results

Each of the four experimental conditions had 30 different participants. Each participant provided one data point. We report how many white marbles they chose to put into the urn. The outcomes of the lotteries are irrelevant to our study and so we do not report them.

Figure 2 shows a box and whiskers plot for the number of white marbles put into the urn in each experimental condition. The colored asterisks mark the optimal number of white marbles that should be put into the urn to maximize expected gain in each condition. For each of the four conditions we used a single-sample t-test with the null hypothesis set to the optimal number of marbles for the given condition. The stars indicate the level of significance (see Figure 2 caption).

The number of white marbles that participants, on average, tended to put into the urn increased systematically with an increase in the number of black marbles in the urn. Furthermore the results indicate that, on average, participants systematically put one more white marble into the urn than dictated by the normative rule that maximizes expected gain (i.e., people, on average, are sub-optimal with respect to the normative rule, preferring a $1 decrease to their potential prize in exchange for an increase to their probability of winning the lottery).

Discussion

Our results seem to indicate that participants, on average, did not maximize expected gain. Curiously though, the number of white marbles that they tended to put into the urn was one more than the normative ideal rule irrespective of the experimental condition. This systematic tendency led us to explore possible explanations to account for their sub-optimal manipulation of the lottery's parameters.

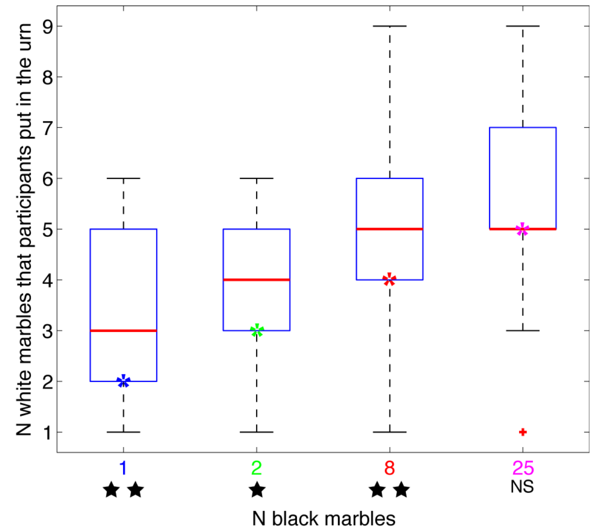

Figure 2: Box and whiskers plot showing the results in all four conditions. The colored asterisks mark the corresponding optimal number of white marbles that should be put into the urn to maximize expected gain. Participants, on average, tended to put "one too many" white marbles into the urn with respect to optimal. Two stars indicate that this was significant at the .01 level, while one star indicates that it was significant at the .05 level. The non-significant condition had a $p = .08$.

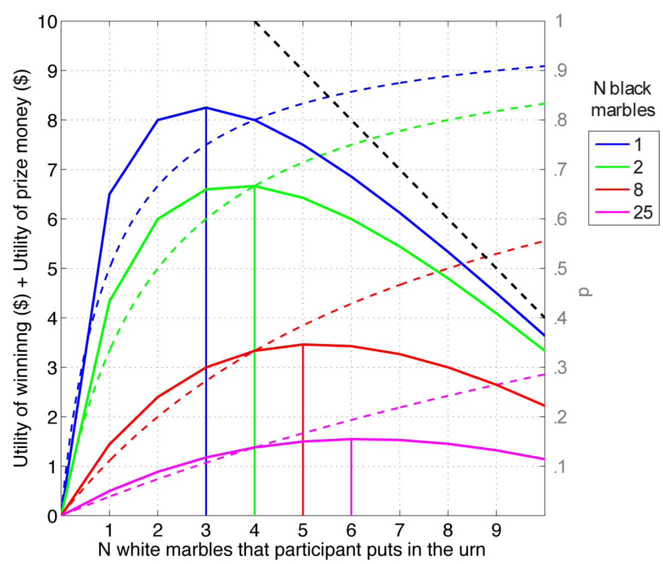

Figure 3: Expected utility functions if we take into account an additional utility of $4 for winning the lottery. Compare this figure to Figure 1 that shows the expected gain functions without taking into account any additional utility of winning. Notice that the maximum expected utilities under this scheme are shifted one marble to the right in all four conditions relative to the maximum expected gains in Figure 1.

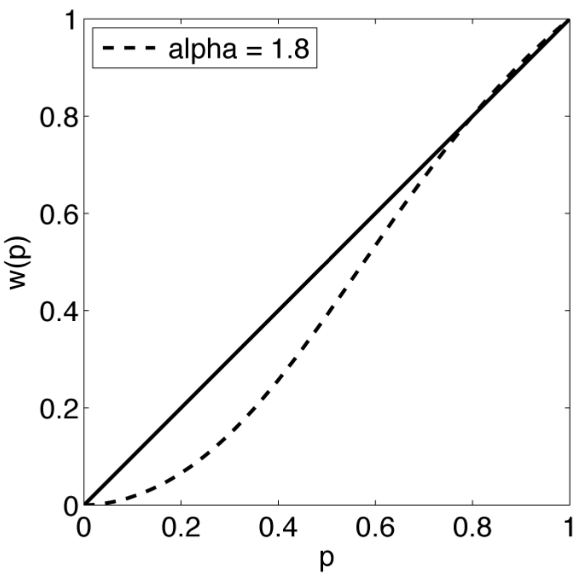

Figure 4: S-shaped probability distortion.

The first possibility we explored is that participants might have had an intrinsic utility for winning the lottery in addition to their utility for the actual money that they receive if they win (Parco, Rapoport, & Amaldoss, 2005). We explored this possibility by putting a fixed dollar value on the intrinsic utility of winning (a free parameter).

Figure 3 shows the expected utility functions when the intrinsic utility of winning is valued at \$4. Notice that the maximum expected utilities shift rightward one marble relative to the normative rule that maximizes expected gain. This rightward shift persists if the intrinsic utility of winning is anywhere between \$3.10 and \$4.60.

This analysis suggests that participants' sub-optimal behavior could be accounted for if they have an intrinsic utility for winning the lottery that is in addition to their utility for the actual money that they receive if they win.

The second possibility we explored is that participants might have had a distortion in subjective probability. A standard single-parameter model for distortion of probability in decision-making under risk is written as follows (Tversky & Kahneman, 1992):

$$w(p) = \frac{p^{\alpha}}{(p^{\alpha} + (1-p)^{\alpha})^{1/\alpha}} \quad (1)$$

As our task resembles a decision from description more than a decision from experience, we hypothesized that an Inverse-S-shaped probability distortion might be more likely to account for the data than an S-shaped probability distortion (Ungemach, Chater, & Stewart, 2009; Wu, Delgado, & Maloney, 2009). In other words, we expected that participants' average behavior might be accounted for with an $\alpha < 1$ as is commonly found in decisions from description, and not with an $\alpha > 1$ as is commonly found in decisions from experience (but see Glaser, Trommershäuser, Mamassian, & Maloney,

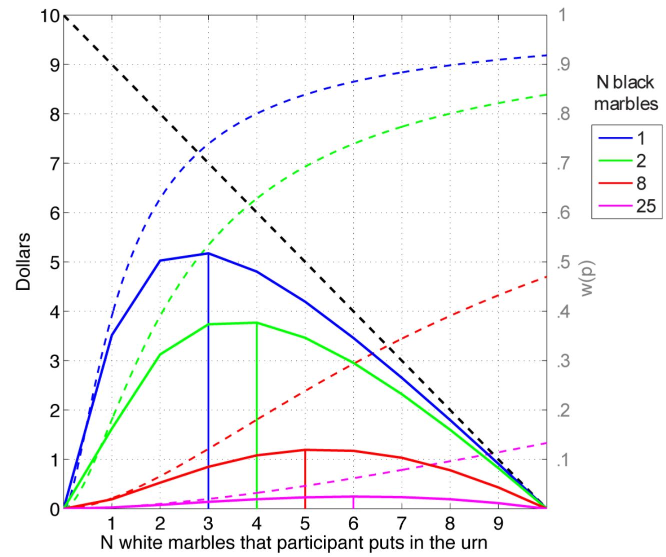

Figure 5: Expected gains based on the S-shaped probability distortion depicted in Figure 4. Compare this figure to Figure 1 that shows the true probabilities and expected gain curves. Notice that the maximum expected gains under this scheme are shifted one marble to the right in all four conditions relative to the maximum expected gains in Figure 1.

2012). However, the only way to shift the maximum expected gain one marble to the right in all four conditions is by having $1.69 < \alpha < 1.92$.

Figure 4 shows an S-shaped probability distortion with $\alpha = 1.8$. Figure 5 shows the consequent expected gain functions based on this distorted probability function with the maximum expected gains shifted one marble to the right.

This analysis suggests that participants' sub-optimal behavior could be accounted for if they have an S-shaped probability distortion, a form only rarely encountered in decisions from description tasks.

Conclusion

Participants in typical decision making experiments select among a small number of lotteries each with fixed probabilities and rewards. In the experiment reported here we considered a task where participants could manipulate their environment to improve their chances of winning the lottery but only by reducing the amount of money that they could possibly win.

While a simple and preliminary study, our results may prove useful to researchers interested in how people balance risk and reward in simple, intuitive decision tasks. Our experiment is unique in a couple of ways that are worth pointing out.

First, participants could actively manipulate the odds of successfully winning the lottery (by placing more or less white marbles into the urn). While the literature on risky decision making is immense, we are unaware of previous studies that considered this *active* manipulation of the decision environment (one exception is a related study we reported last year in Juni et al., 2011).

Second, the decision environment was set up to be intuitive, fast, and easy to conduct with everyday people walking through a park. As a result, we collected a more diverse sample than is typical for research on judgment and decision-making.

Our primary results are that participants, on average, did not maximize expected gain. In particular, participants, on average, tended to put one additional white marble into the urn than dictated by the normative ideal rule. By doing so they slightly increased their probability of winning relative to ideal, but also decreased their expected winnings slightly.

We considered two different accounts for participants' sub-optimal choice behavior. Participants in decisions from description typically over-weight small probabilities and under-weight large probabilities (Inverse-S-shaped probability distortion). However, we found that we could account for the results obtained only if we assumed that participants under-weighted small probabilities and over-weighted large probabilities (S-shaped probability distortion). While such a tendency has been found in decisions from experience, our task resembles a decision from description more than a decision from experience, making an S-shaped probability distortion surprising.

A second account for our data is that participants have an intrinsic utility for winning the lottery that is in addition to their utility for the actual money that they receive if they win. If this account of participants' behavior is correct, it would seem that their intrinsic utility of winning is approximately $4.

Future studies could tease apart these two accounts of participants' behavior by keeping the objective probabilities of the lotteries the same and simply scaling up the value of each white marble. According to the probability distortion account we should see no change in participants' average behavior. But according to the intrinsic utility of winning account we should see participants' average behavior shift toward optimal as the intrinsic utility of winning is diluted relative to the increased potential winnings that the lotteries afford.

Acknowledgments

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